tions of the quadratic correspond to having P and Q on the same, or on opposite, sides of the reference triangle ABC. This application is convenient for triangulation in three dimensions using only distances and is independent of coordinate axes.

The second use is for solving the problem propounded above. Suppose the radius of the fifth sphere, inscribed between spheres 1, 2, 3 and 4, is R, then the determinant relating the ten distances becomes:

$$D(R) = \begin{vmatrix} 0 & d_{12}^2 & d_{13}^2 & d_{14}^2 & (r_1+R)^2 & 1 \\ d_{12}^2 & 0 & d_{23}^2 & d_{24}^2 & (r_2+R)^2 & 1 \\ d_{13}^2 & d_{23}^2 & 0 & d_{34}^2 & (r_3+R)^2 & 1 \\ d_{14}^2 & d_{24}^2 & d_{34}^2 & 0 & (r_4+R)^2 & 1 \\ (r_1+R)^2 & (r_2+R)^2 & (r_3+R)^2 & (r_4+R)^2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{vmatrix} = 0 .$$

The problem thus reduces to one of finding the appropriate zero of $D(R)$. This is readily computed with a subroutine for the evaluation of a determinant by pivotal condensation. R is increased by small steps from zero. When $D(R)$ changes sign we take one step back and advance by smaller steps and so on until R is known to the required accuracy.

The negative root corresponds to the circumscribing sphere.

The basic formula used above for the volume of a simplex can be proved by reducing the determinant of order 6 (rank 4) to one of order 4 by subtracting the first row from each of the other rows and doing similarly with the first column. If the four vectors corresponding to d_{12}, d_{13}, d_{14} and d_{15} are written **a**, **b**, **c** and **d** then, since $(\mathbf{a}-\mathbf{b})^2-\mathbf{a}^2-\mathbf{b}^2 = -2\,\mathbf{a}\,.\,\mathbf{b}$, the determinant becomes

$$-(-2)^4 = \begin{vmatrix} \mathbf{a}^2 & \mathbf{a}\,.\,\mathbf{b} & \mathbf{a}\,.\,\mathbf{c} & \mathbf{a}\,.\,\mathbf{d} \\ \mathbf{a}\,.\,\mathbf{b} & \mathbf{b}^2 & \mathbf{b}\,.\,\mathbf{c} & \mathbf{b}\,.\,\mathbf{d} \\ \mathbf{a}\,.\,\mathbf{c} & \mathbf{b}\,.\,\mathbf{c} & \mathbf{c}^2 & \mathbf{c}\,.\,\mathbf{d} \\ \mathbf{a}\,.\,\mathbf{d} & \mathbf{b}\,.\,\mathbf{d} & \mathbf{c}\,.\,\mathbf{d} & \mathbf{d}^2 \end{vmatrix} .$$

This is the determinant of the metric matrix and gives the square of the four-dimensional volume of the parallelepiped (measure polytope) outlined by the vectors **a**, **b**, **c** and **d**. The volume of the simplex is found from that of the parallelepiped by dividing by 4!.

Reference

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Corrections to the Tables in Chapter 5.1, *Reduced Cells*, given in the 1969 edition of Volume I of *International Tables*.

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Corrections are given to Table 5.1.2.2 of *International Tables for X-ray Crystallography*, Vol. I (1969), Birmingham: Kynoch Press)

Certain errors have been found in *International Tables for X-ray Crystallography* (1969) in addition to those already pointed out by Mighell, Santoro & Donnay (1971) and the corrections are given below.

Table 5.1.2.2 on page 532

Fourth matrix row from top of table:

Replace matrix S′ $\begin{pmatrix} \mathbf{a}\,.\,\mathbf{a} & \mathbf{b}\,.\,\mathbf{b} & \mathbf{c}\,.\,\mathbf{c} \\ \mathbf{a}\,.\,\mathbf{b}-\mathbf{a}\,.\,\mathbf{c} & \dfrac{\mathbf{a}\,.\,\mathbf{a}}{2} & \mathbf{a}\,.\,\mathbf{b} \end{pmatrix}$

by: $\begin{pmatrix} \mathbf{a}\,.\,\mathbf{a} & \mathbf{b}\,.\,\mathbf{b} & \mathbf{c}\,.\,\mathbf{c} \\ \mathbf{a}\,.\,\mathbf{b}-\mathbf{b}\,.\,\mathbf{c} & \dfrac{\mathbf{a}\,.\,\mathbf{a}}{2} & \mathbf{a}\,.\,\mathbf{b} \end{pmatrix}$

Bottom matrix row of table:

Replace matrix S $\begin{pmatrix} \mathbf{a}\,.\,\mathbf{a} & \mathbf{b}\,.\,\mathbf{b} & \mathbf{c}\,.\,\mathbf{c} \\ \mathrm{X} & \mathbf{a}\,.\,\mathbf{c} & \mathbf{a}\,.\,\mathbf{b} \end{pmatrix}$

by: $\begin{pmatrix} \mathbf{a}\,.\,\mathbf{a} & \mathbf{b}\,.\,\mathbf{b} & \mathbf{c}\,.\,\mathbf{c} \\ -\mathrm{X} & \mathbf{a}\,.\,\mathbf{c} & \mathbf{a}\,.\,\mathbf{b} \end{pmatrix}$

References

International Tables for X-ray Crystallography (1969). Vol. I. Birmingham: Kynoch Press.

MIGHELL, A. D., SANTORO, A. & DONNAY, J. D. H. (1971). *Acta Cryst.* B**27**, 1837–1838.